Exercise 38

(a) If $f(x) = e^x/(2x^2 + x + 1)$, find $f'(x)$.

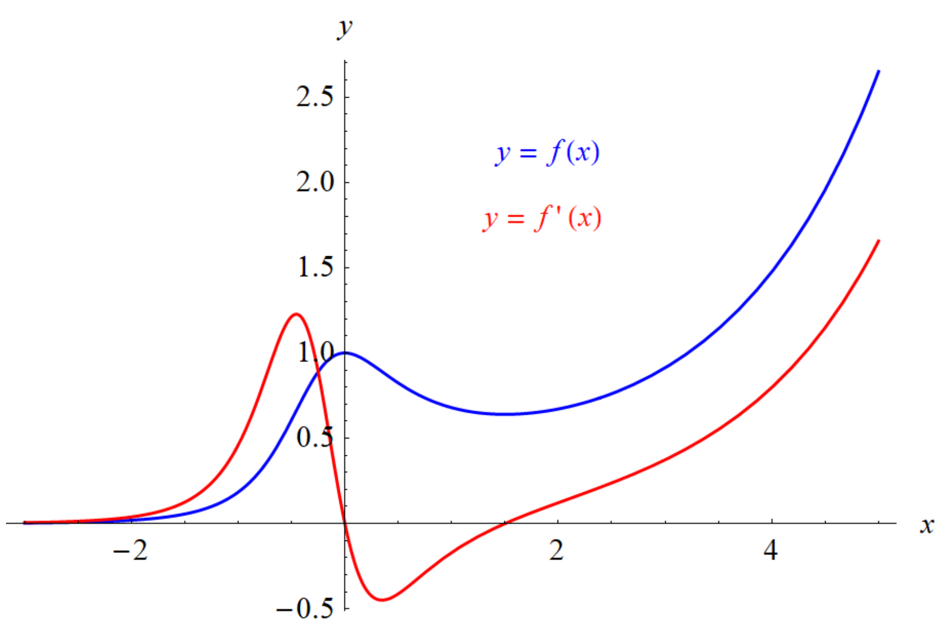
(b) Check to see that your answer to part (a) is reasonable by comparing the graphs of f and f'.

Solution

Evaluate the derivative using the quotient rule.

$$\begin{aligned}
f'(x) &= \frac{d}{dx}\left(\frac{e^x}{2x^2 + x + 1}\right) \\
&= \frac{\left[\frac{d}{dx}(e^x)\right](2x^2 + x + 1) - \left[\frac{d}{dx}(2x^2 + x + 1)\right](e^x)}{(2x^2 + x + 1)^2} \\
&= \frac{(e^x)(2x^2 + x + 1) - (4x + 1)(e^x)}{(2x^2 + x + 1)^2} \\
&= \frac{(2x^2 - 3x)e^x}{(2x^2 + x + 1)^2} \\
&= \frac{x(2x - 3)e^x}{(2x^2 + x + 1)^2}
\end{aligned}$$

Below is a graph of the function and its derivative versus x.



$f'(x)$ is positive wherever $f(x)$ increases, $f'(x)$ is zero wherever the slope of $f(x)$ is zero, and $f'(x)$ is negative wherever $f(x)$ is decreasing. The answer to part (a) is reasonable then.